1. A box contains 10 red, 20 white and 30 blue beads. After a thorough shaking, a blindfolded person draws out a bead. What is the probability that (a) it is blue? (b) red or blue? (c) neither red or blue?

2. Say he draws out 2 beads, returning the bead after the 1st draw. What is the probability that (a) he will draw two blues? (b) he will draw first a white, then a red? (c) he will draw a red and a white?

3. An engineer is to make up test specimens consisting of an assembly of parts A and B. Part A is chosen from a lot that is known to be 6% defective and part B from a 4% defective lot. What is the probability that his test specimen will be defective if he considers a defective assembly as one having either/or both parts defective?

4. A disease has a mortality rate of 80%. In testing a particular drug, a physician finds that 4 patients recover out of 5 on whom the drug is tried. Compute the probability of obtaining results as good as these assuming the drug is ineffective.

5. a. Find the probability of hitting a ship when 4 torpedoes are fired if each torpedo has a probability of 1/4 of hitting. (Ans. .68)

   b. How many torpedoes should be fired to raise the probability of a hit to .76. (Ans. 5)

6. A, B, and C each fire one shot at a target. The probabilities of each hitting the target are:

   A: 0.30
   B: 0.25
   C: 0.10

   If one bullet is found in the target, find the probability that it came from A's gun; from B's gun; from C's gun.

H M M M	1
H H M M	2
H H H M	3
M H H H	3
H H H H	4
M M M M	0

1. A TV repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

(Ans. 3 hours; 1-2/3 jobs)

2. At what average rate must a clerk at a supermarket work in order to insure a probability of 0.90 that the customer will not have to wait longer than 12 minutes? It is assumed that there is only one counter, to which customers arrive in a Poisson fashion at an average rate of 15 per hour. The length of service by the clerk has an exponential distribution.

(Ans. 2.26 minutes per service)

3. A supermarket has two girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes, and if people arrive in a Poisson fashion at the counter at the rate of 10 an hour:

(a) What is the probability of having to wait for service?

(Ans. 0.167)

(b) What is the expected percentage of idle time for each girl?

(Ans. 67%)

4. A telephone exchange has two long-distance operators. The telephone company finds that, during the peak load, long-distance calls arrive in a Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes. What is the probability that a subscriber will have to wait for his long-distance call during the peak hours of the day? If the subscribers will wait and are serviced in turn, what is the expected waiting time?

(Ans. 0.48, 3.2 minutes)

5. A bank has two tellers working on savings accounts. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time distributions for both deposits and withdrawals are exponential with mean service time 3 minutes per customer. Depositors are found to arrive in a Poisson fashion through-out the day with mean arrival rate 16 per hour. Withdrawers also arrive in a Poisson fashion with mean arrival rate 14 per hour. What would be the effect on the average waiting time for depositors and withdrawers if each teller could handle both withdrawals and deposits? What would be the effect if this could only be accomplished by increasing the mean service time to 3.5 minutes?

(Ans. Initially 12 minutes and 7 minutes. Combined waiting time 3.86 minutes. Combined waiting time with increased service time 11.4 minutes.)

6. At a certain airport it takes exactly 5 minutes to land an airplane, once it is given the signal to land. Although incoming planes have scheduled arrival times, the wide variability in arrival times produces an effect which makes the incoming planes appear to arrive in a Poisson fashion at an average rate of six per hour. This produces occasional stack-ups at the airport which can be dangerous and costly. Under these circumstances, how much time will a pilot expect to spend circling the field waiting to land?

(Ans. 2-1/2 minutes)

7. Problems arrive at a computing center in a Poisson fashion at an average rate of five per day. The rules of the computing center are that any man waiting to get his problem solved must aid the man whose problem is being solved. If the time to solve a problem with one man has an exponential distribution with a mean time of 1/3 day, and if the average solving time is inversely proportional to the number of people working on the problem, approximate the expected time in the center for a person entering the line.

(Ans. 8 hours)

8. A warehouse in a small state receives orders for a certain item and sends them by truck as soon as possible to the customer. The orders arrive in a Poisson fashion at a mean rate of 0.9 per day. Only one item at a time can be shipped by truck from the warehouse, which is located in the central part of the state. Because the customers are located in various places in the state, the distribution of service time in days has a distribution with probability density $4te^{-2t}$. What is the expected delay between the arrival of an order and the arrival of the item to the customer? Service time is defined here as the time the truck takes to load, get to the customer, unload, and return to the warehouse. Loading and unloading times are small compared with travel time.

(Ans. 7-1/4 days)

9. A hospital clinic has a doctor examining every patient brought in for a general checkup. The doctor averages 4 minutes on each phase of the checkup although the distribution of time spent on each phase is approximately exponential. If each patient goes through four phases in the checkup and if the arrivals of the patients to the doctor's office are approximately Poisson at an average rate of three per hour, what is the average time spent by a

patient waiting in the doctor's office? What is the average time spent in the examination? What is the most probable time spent in the examination?

(Ans. 40 minutes, 16 minutes, 12 minutes)

10. Before parts are assembled into a vacuum tube, they must be cleaned in a degreaser. Batches of parts are brought in randomly (Poisson) at an average rate of λ batches per hour, and are cleaned at an average rate of μ batches per hour. The cost of waiting is C_1 dollars per batch per hour; and the cost of owning and operating a degreaser that works at an average rate μ is μC_2 dollars per hour. How large should μ be?

(Ans. For least expected hourly cost,

$$\mu = \lambda + \sqrt{C_1 \lambda / C_2}$$

1. A baking company sells cake by the pound. It makes a profit of 50 cents a pound on every pound sold on the day it is baked. It disposes of all cake not sold on the date it is baked at a loss of 12 cents a pound. If demand is known to be rectangular between 2000 and 3000 pounds, determine the optimal daily amount baked.

(Ans. 2807 pounds)

2. The probability distribution of monthly sales of a certain item is as follows:

Monthly Sales	Probability
0	0.02
1	0.05
2	0.30
3	0.27
4	0.20
5.	0.10
6	0.06
	1.00

The cost of carrying inventory is $10 per unit per month. The current policy is to maintain a stock of four items at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity short, obtain the imputed cost of a shortage of one item for one time unit. (Because the problem is stated in discrete units, the answer will consist of a range of values for the imputed cost.)

3. A newspaper boy buys papers for 3 cents each and sells them for 7 cents each. He cannot return unsold newspapers. Daily demand has the following distribution:

No. of customers	23	24	25	26	27	28	29	30	31	32
Probability	.01	.03	.06	.10	.20	.25	.15	.10	.05	.05

If each day's demand is independent of the previous day's, how many papers should he order each day?

4. A manufacturer of a perishable commodity is limited, because he takes advantage of bulk transportation, to one shipment a month from factory to warehouse. This shipment takes place on the first of each month, and includes all the previous month's production. There is one production run a month, which starts at the beginning of the month and continues at the rate of k units per day until the monthly production quota of q units is finished. (Here k is fixed, and q is a decision variable.) Production costs C per unit.

Goods are sold only from the warehouse. The probability of a demand on the warehouse for a quantity n during the month is p(n); it may be assumed that the demand on the warehouse is spread uniformly over the month. Storage, either at the warehouse or at the factory, costs C_1 per unit per month. Any warehouse surplus at the end of the month is considered stale, inasmuch as it is over a month old, and is disposed of through outlets which will take an unlimited amount of stale goods, but at a price P' (less than C) per unit. The sales price of fresh goods (i. e. goods received at the warehouse on the first of the current month) is P per unit.

Find the expected profit per month as a function of the monthly production quota q, and, by differencing this function, determine the condition for optimal q.

5. A company owns two warehouses X and Y. Shipments from the factory are made on the first of each month to bring the stock at X up to S_1 and the stock at Y up to S_2. Demand during the month is uniform at each warehouse, with a density function f(x) at X and g(y) at Y, and with x and y independent. Inventory holding costs are C_1 per unit per month and shortage costs are C_2 per unit per month, at either warehouse. In the event of a shortage at one warehouse, deliveries are made from the other warehouse, but only provided the latter is certain that its own orders for the month can be taken care of out of the stock that remains. However, it costs an extra amount k per unit to make a delivery from X to a customer of Y, and vice versa. It may be assumed that, by the time one warehouse has run out of stock, the other warehouse can predict its total demand for the month exactly.

Show how to find values of S_1 and S_2 which will minimize the total expected cost.

A ship has three cargo holds, forward, aft, and center. The capacity limits are:

Forward	2000 tons	100,000 cubic feet
Center	3000 tons	135,000 cubic feet
Aft	1500 tons	30,000 cubic feet

The following cargoes are offered; the shipowners may accept all or any part of each commodity:

Commodity	Amount, tons	Volume per ton, cu ft	Profit per ton,$
A	6,000	60	6
B	4,000	50	8
C	2,000	25	5

In order to preserve the trim of the ship, the weight in each hold must be proportional to the capacity in tons. How should the cargo be distributed so as to maximize profit? (Note: Since the restrictions will contain an equality, the approach suggested in the previous problem may be needed in order to obtain a numerical answer.)

* * * \

		To item				
		A	B	C	D	E
From item	A	∞	4	7	3	4
	B	4	∞	6	3	4
	C	7	6	∞	7	5
	D	3	3	7	∞	7
	E	4	4	5	7	∞

In addition, there is a priority rating among the items, A and B having a higher priority than C, D, E. This may be interpreted as adding an additional cost of 5 when any high-priority item immediately follows a low-priority item. If he processes each type of item once and only once each week, how should he sequence the items on his machine?